Cascade-Dissipation Balance in Space Plasma Turbulence

Insights from the Terrestrial Magnetosheath

PNST 8-12 Jan 2024 - Marseille

LPP
Laboratoire de Physique des Plasmas

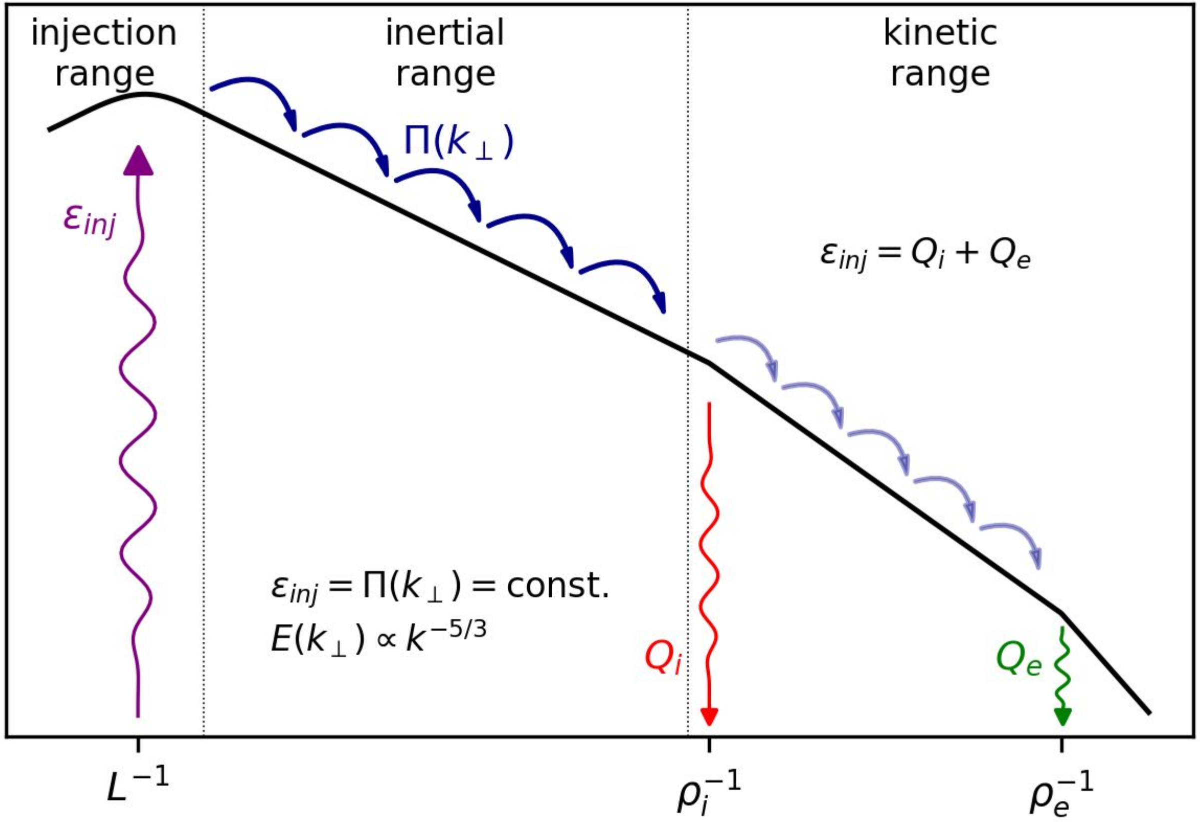
Davide Manzini[1,2], F. Sahraoui[1], F. Califano[2].

[1]Laboratoire de Physique des Plasmas, École Polytechnique, France

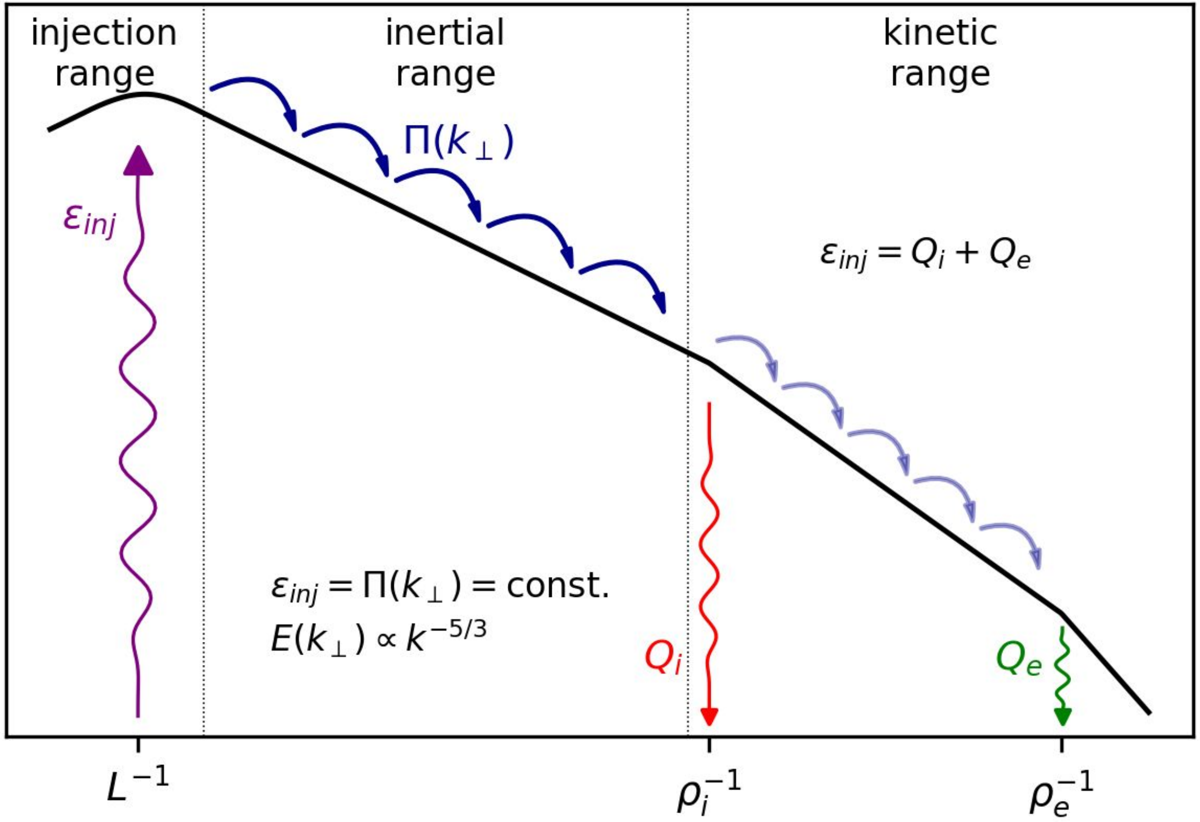
[2]Dipartimento di Fisica E.Fermi, Universitá di Pisa, Italia

cnrs
École Polytechnique
Institut Polytechnique de Paris
Sorbonne Université
Université Paris-Saclay
l'Observatoire de Paris
Università di Pisa

Cascade and Dissipation in Plasmas

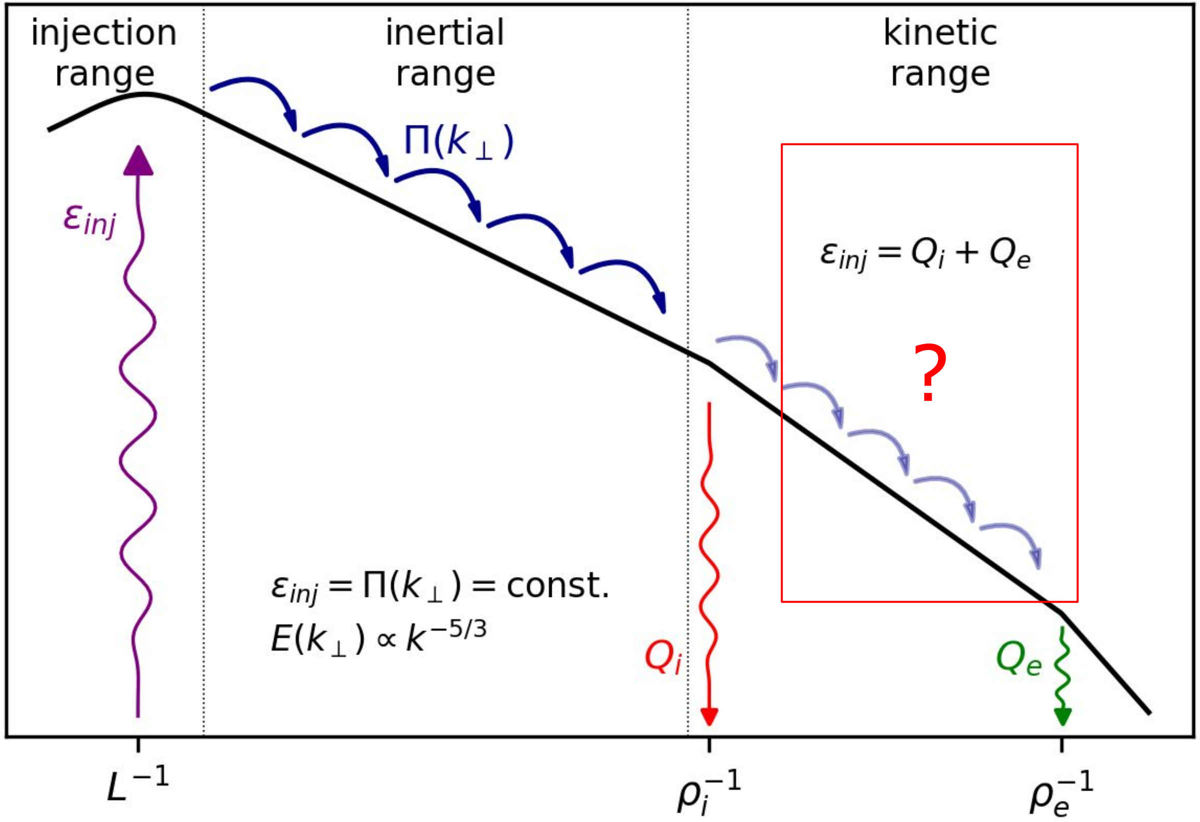


Cascade and Dissipation in Plasmas



Q_1: How is the energy cascade modified by plasma phenomena (e.g. MR)?

Cascade and Dissipation in Plasmas



Q_1: How is the energy cascade modified by plasma phenomena (e.g. MR)?

Q_2: What happens at the cascade in the kinetic range?

And ultimately:
what sets Q_i/Q_e?

The Cascade Rate

$$\Pi(K) = -\frac{\partial}{\partial t}\int_{|\mathbf{k}'|<K} E(\mathbf{k}')d\mathbf{k}'\Big|_{\mathrm{NL}}$$

Rate of change, due to non-linearities, of large scale energy.

In the *inertial range* we compute the cascade rate using third order structure functions:

$$\Pi \sim -\frac{4}{3\ell}\left\langle (|\delta\mathbf{v}|^2 + |\delta\mathbf{b}|^2)\delta v_\ell - 2(\delta\mathbf{v}\cdot\delta\mathbf{b})\delta b_\ell \right\rangle$$

The Cascade Rate

$$\Pi(K) = -\frac{\partial}{\partial t} \int_{|\mathbf{k}'|<K} E(\mathbf{k}')d\mathbf{k}' \Big|_{\mathrm{NL}}$$

Rate of change, due to non-linearities, of large scale energy.

In the *inertial range* we compute the cascade rate using third order structure functions:

$$\Pi \sim -\frac{4}{3\ell}\left\langle (|\delta \mathbf{v}|^2 + |\delta \mathbf{b}|^2)\delta v_\ell - 2(\delta \mathbf{v} \cdot \delta \mathbf{b})\delta b_\ell \right\rangle$$

Statistical convergence is an issue! In the SW we need one year of data for a reliable estimate [Stawarz+ 2009 *ApJ*]

The Cascade Rate

$$\Pi(K) = -\frac{\partial}{\partial t} \int_{|\mathbf{k}'|<K} E(\mathbf{k}') d\mathbf{k}' \Big|_{\mathrm{NL}}$$

Rate of change, due to non-linearities, of large scale energy.

In the *inertial range* we compute the cascade rate using third order structure functions:

$$\Pi \sim -\frac{4}{3\ell} \left\langle (|\delta \mathbf{v}|^2 + |\delta \mathbf{b}|^2)\delta v_\ell - 2(\delta \mathbf{v} \cdot \delta \mathbf{b})\delta b_\ell \right\rangle$$

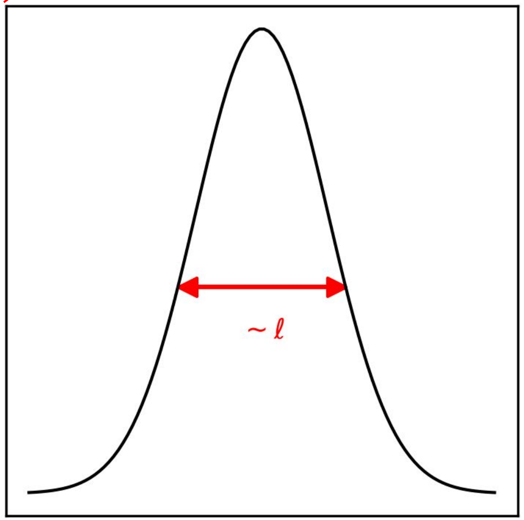
Statistical convergence is an issue! In the SW we need one year of data for a reliable estimate [Stawarz+ 2009 *ApJ*]

Can we overcome this limitation?

An alternative formulation: the Coarse-Graining approach

The coarse-graining operation is a local average of the field

$$\bar{\mathbf{u}}_\ell = \mathbf{u} * G_\ell = \int \mathbf{u}(\mathbf{x}+\mathbf{x}')G_\ell(\mathbf{x}')d\mathbf{x}'$$



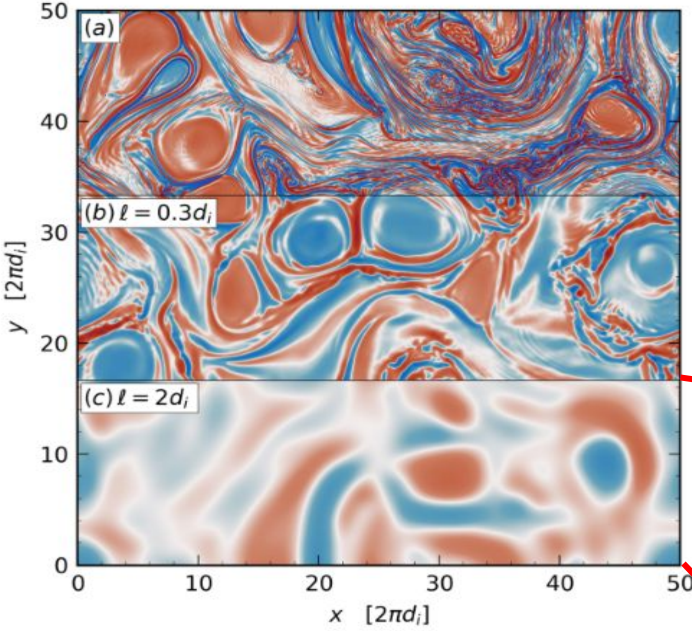
An alternative formulation: the Coarse-Graining approach

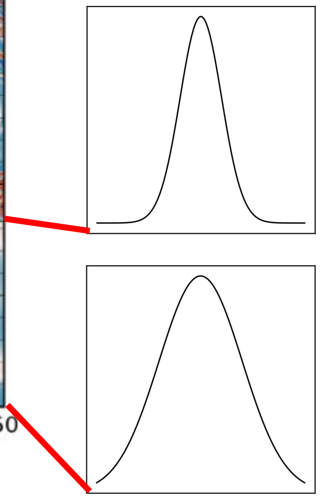
The coarse-graining operation is a local average of the field

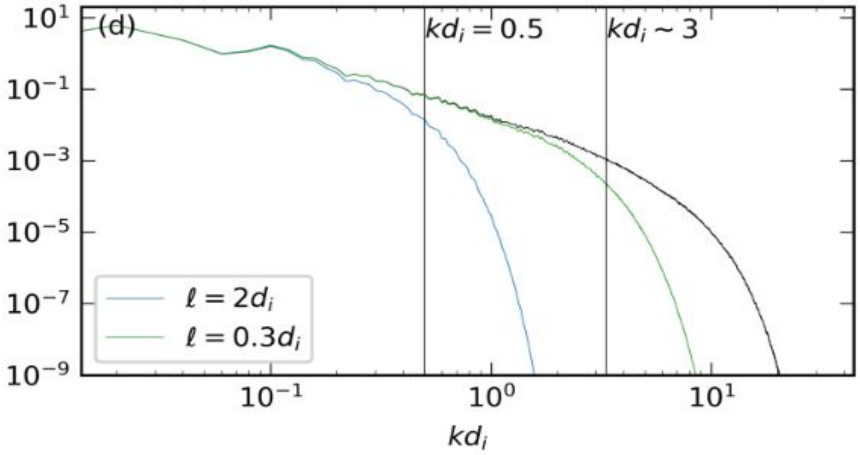
$$\bar{\mathbf{u}}_\ell = \mathbf{u} * G_\ell = \int \mathbf{u}(\mathbf{x}+\mathbf{x}')G_\ell(\mathbf{x}')d\mathbf{x}'$$

We only retain "large scale" components $|\mathbf{k}| \lesssim 1/\ell$

$$\mathrm{PSD}\{\bar{\mathbf{u}}_\ell\} = |\hat{\mathbf{u}}(\mathbf{k})|^2 |\hat{G}_\ell(\mathbf{k})|^2$$







An alternative formulation: the Coarse-Graining approach

The method pioneered by G. L. Eyink[1] and H. Aluie[2] enables us to write equations for large scale energies.

E.g. in the framework of Incompressible Hall MHD[3,4] and for each scale ℓ

$$\frac{\partial}{\partial t}\left(\frac{|\bar{\mathbf{u}}_\ell|^2+|\bar{\mathbf{b}}_\ell|^2}{2}\right)+\nabla\cdot\mathcal{F}_\ell=\bar{\mathbf{u}}_\ell\cdot\bar{\mathbf{d}}_\ell^\nu+\bar{\mathbf{b}}_\ell\cdot\bar{\mathbf{d}}_\ell^\eta+\bar{\mathbf{u}}_\ell\cdot\bar{\mathbf{f}}_\ell-\pi_\ell$$

LS Energy density

Spatial Fluxes

LS Dissipation

LS Forcing

Non-linear transfer to smaller scales

[1]Eyink, Physica D (2005) [2]Eyink & Aluie PoF (2009)
[3]Camporeale PRL (2018) [4]Manzini PRE (2022)

The "local cascade rate"

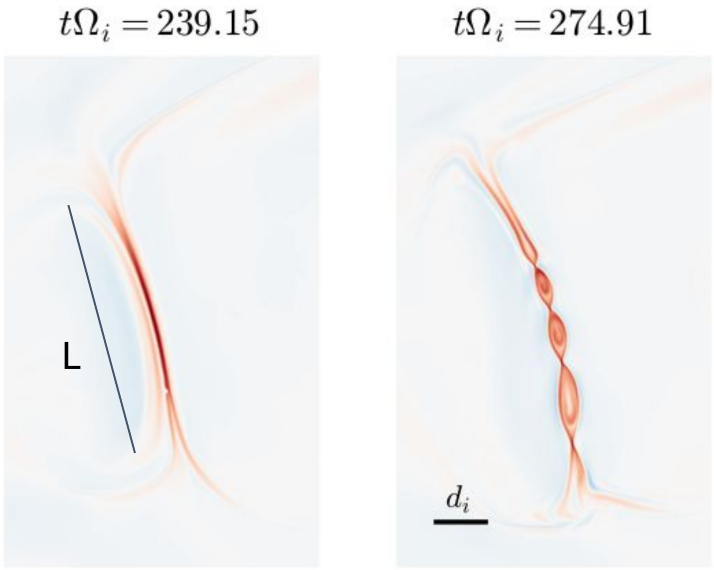
Change of large scale energy density due to non-linear interactions

Cross-scale transfer across scale ℓ "local cascade rate"

$$-\frac{\partial}{\partial t}\left(\rho_0 \frac{|\bar{\mathbf{u}}_\ell|^2 + |\bar{\mathbf{b}}_\ell|^2}{2}\right)\Bigg|_{\mathrm{NL}} = \pi_\ell(x) = -\rho_0 \nabla \bar{\mathbf{u}}_\ell : \boldsymbol{\tau}_\ell - \bar{\mathbf{j}}_\ell \cdot \boldsymbol{\mathcal{E}}_\ell$$

$\pi_\ell(x)$ is the cross-scale transfer across scale ℓ "at position x"

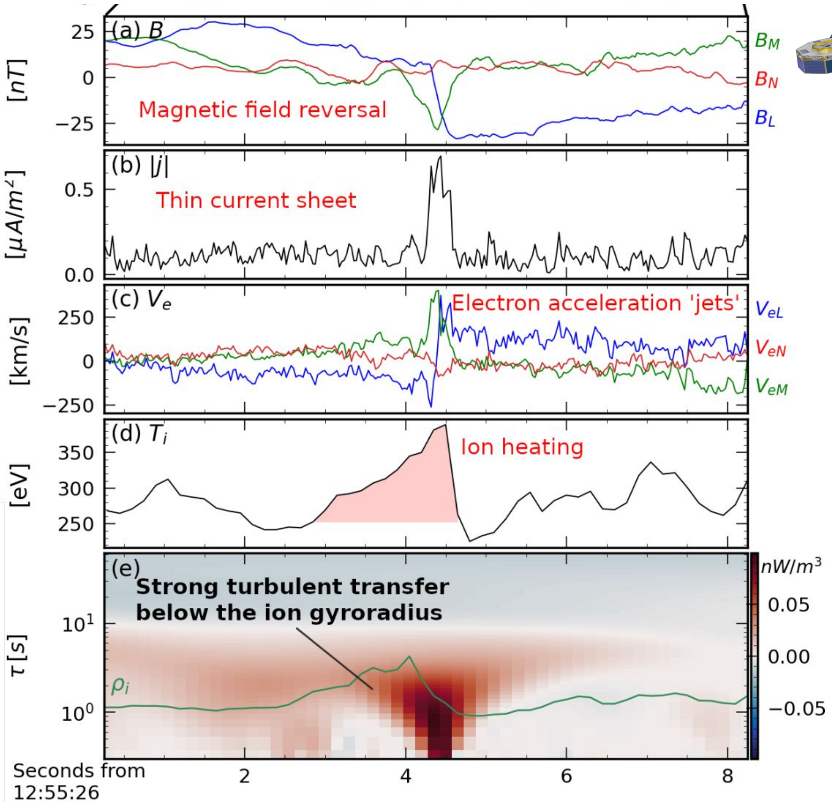
The "local cascade rate" and Magnetic Reconnection

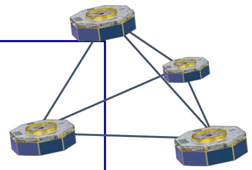


Can we use CG to diagnose *localized cross-scale energy transfer* associated with MR?

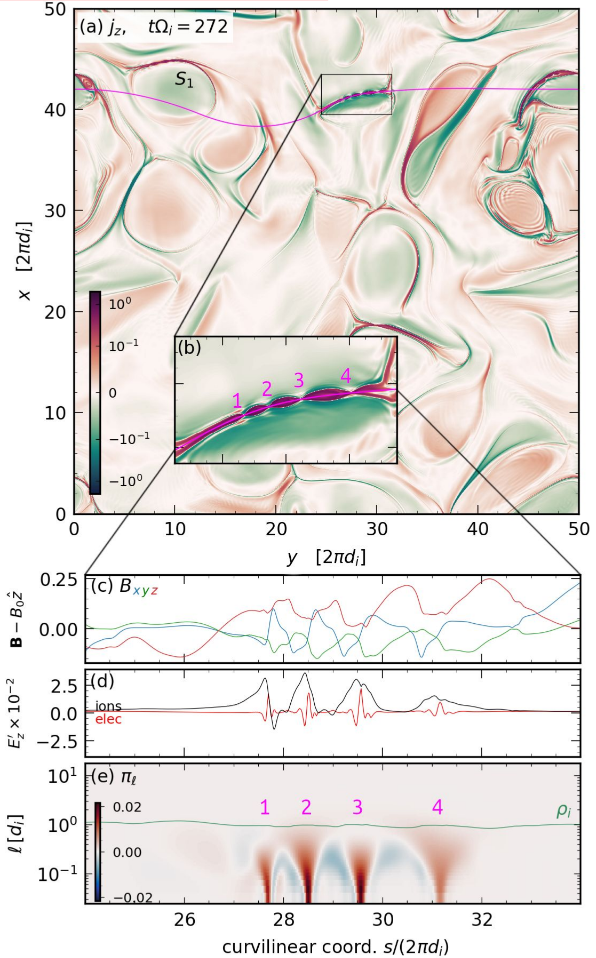
Magnetic Reconnection

MMS data



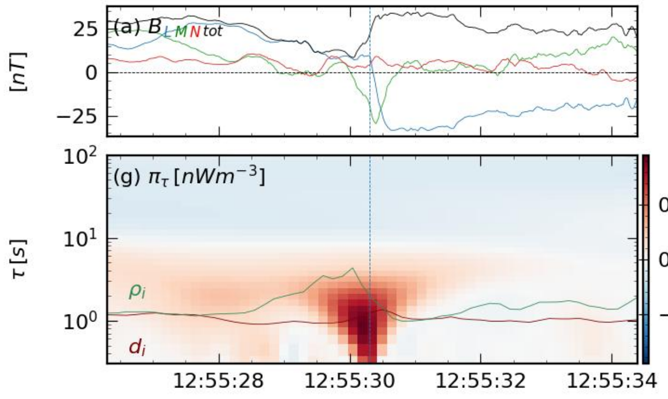


HVM simulations

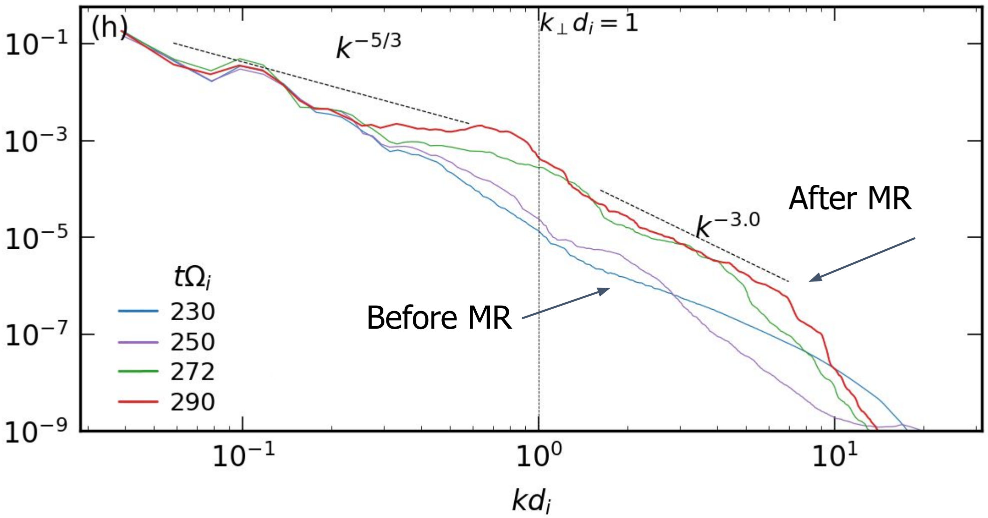


Magnetic Reconnection

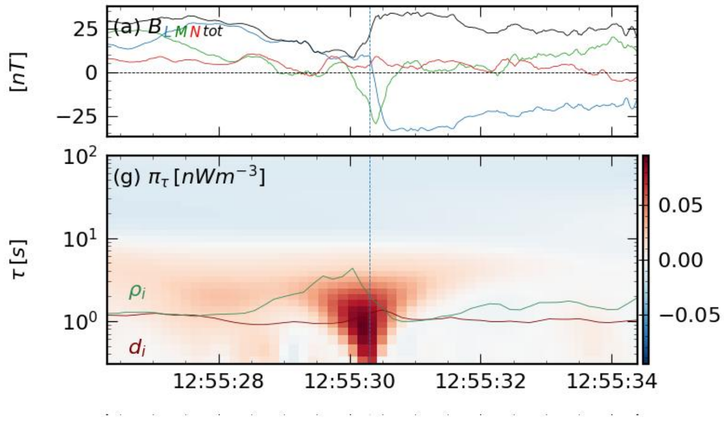
MR drives locally a cross-scale energy transfer to smaller scales
The transfer is effective at subion scale [Manzini+ PRL 2023]



This is reflected in the Magnetic field Spectrum [Franci+ ApjL 2017]

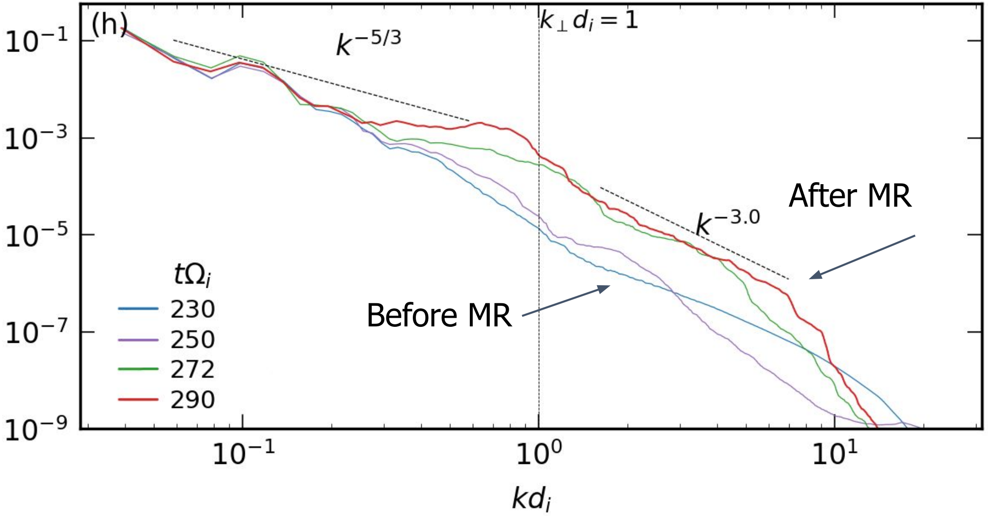


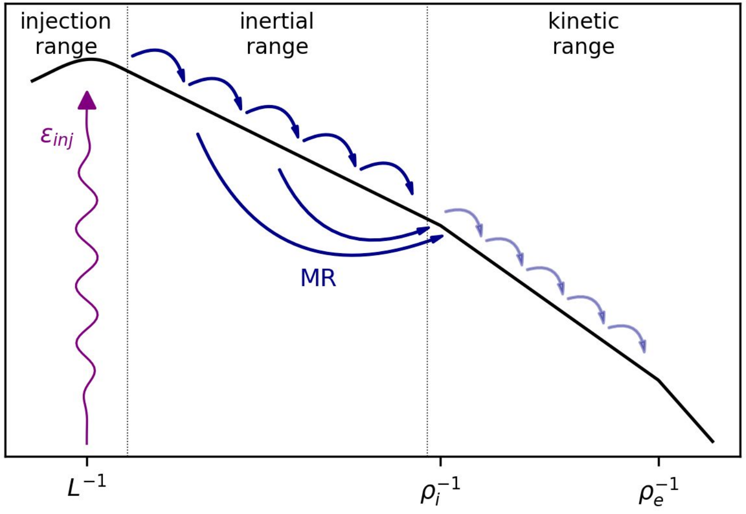
Magnetic Reconnection



MR drives locally a cross-scale energy transfer to smaller scales
The transfer is effective at subion scale [Manzini+ PRL 2023]

This is reflected in the Magnetic field Spectrum [Franci+ ApjL 2017]





Beyond HMHD and the Kinetic range

Particle flow <-> Fields

Large Scale Energies

Particle flow <-> "Thermal En."

$$\frac{\partial}{\partial t}\left\langle \widetilde{\mathcal{E}}^{\mathrm{f}}_p + \widetilde{\mathcal{E}}^{\mathrm{f}}_e \right\rangle = \bar{j}\cdot\bar{E} + \overline{P_p} : \nabla\overline{v_p} + \overline{P_e} : \nabla\overline{v_e} - \Pi_\ell$$

$$\frac{\partial}{\partial t}\left\langle \bar{\mathcal{E}}^{em} \right\rangle = -\bar{j}\cdot\bar{E}$$

Cascade to smaller scales

$$\frac{\partial}{\partial t}\left\langle \bar{\mathcal{E}}^{\mathrm{th}}_\alpha \right\rangle = -\overline{P_\alpha} : \nabla\overline{v_\alpha} - \Phi_\alpha \qquad \alpha = p, e$$

$$\mathcal{E}^f_\alpha = \rho_\alpha |\mathbf{u}_\alpha|^2/2 \quad \mathcal{E}^{em} = (|\mathbf{E}|^2 + |\mathbf{B}|^2)/8\pi \quad \mathcal{E}^{th}_\alpha = \mathrm{Tr}(\mathbf{P}_\alpha)/2$$

Cascade of Kinetic + EM Energy

Large Scale Energies

Transfer to “Thermal En.”

$$\frac{\partial}{\partial t}\left\langle \widetilde{\mathcal{E}}^{\mathrm{f}}_p + \widetilde{\mathcal{E}}^{\mathrm{f}}_e + \bar{\mathcal{E}}^{em} \right\rangle + \nabla\cdot\langle J^{\mathrm{f}}_\ell + J^{\mathrm{em}}_\ell\rangle = \mathrm{PS}_p(\ell) + \mathrm{PS}_e(\ell) - \Pi(\ell)$$

= cst

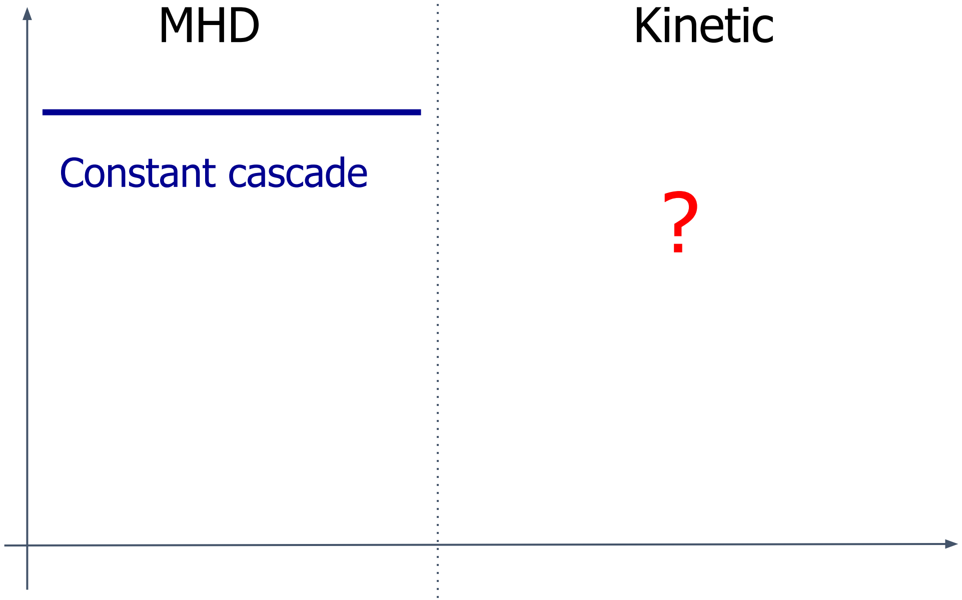
0

Cascade

$$\overline{P_p} : \nabla\overline{v_p} + \overline{P_e} : \nabla\overline{v_e}$$

Cascade of Kinetic + EM Energy

$$\Pi(\ell) - \mathrm{PS}_p(\ell) - \mathrm{PS}_e(\ell) = \mathrm{cst} = \Pi^{MHD}$$



Energy can:
- Continue to cascade
- Heat the plasma via the PS

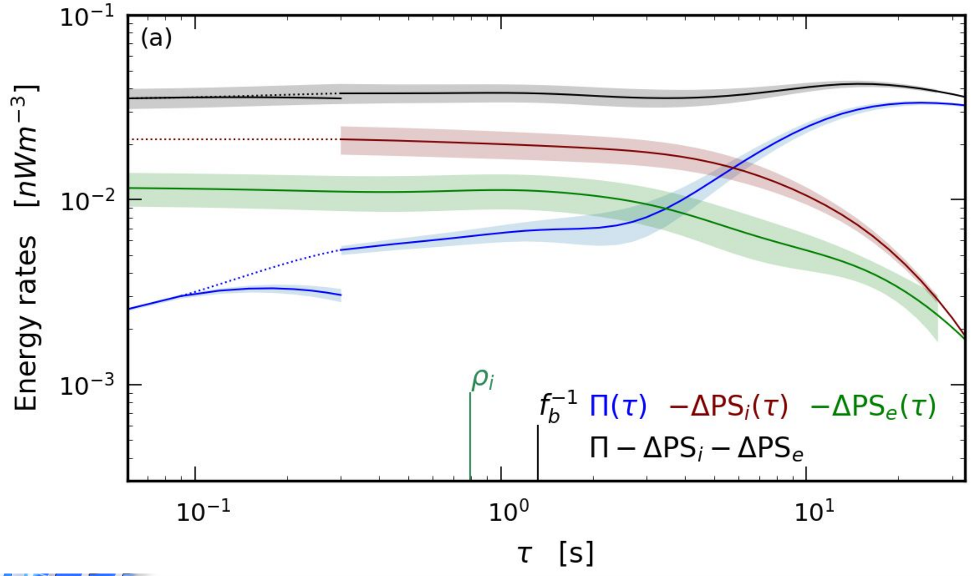
A *scale dependent* cascade implies *dissipation*

Cascade of Kinetic + EM Energy **in MMS data** [Manzini *submitted* (2023)]

$$\mathrm{PS}_p(\ell) + \mathrm{PS}_e(\ell) - \Pi(\ell) = \text{cst.}$$

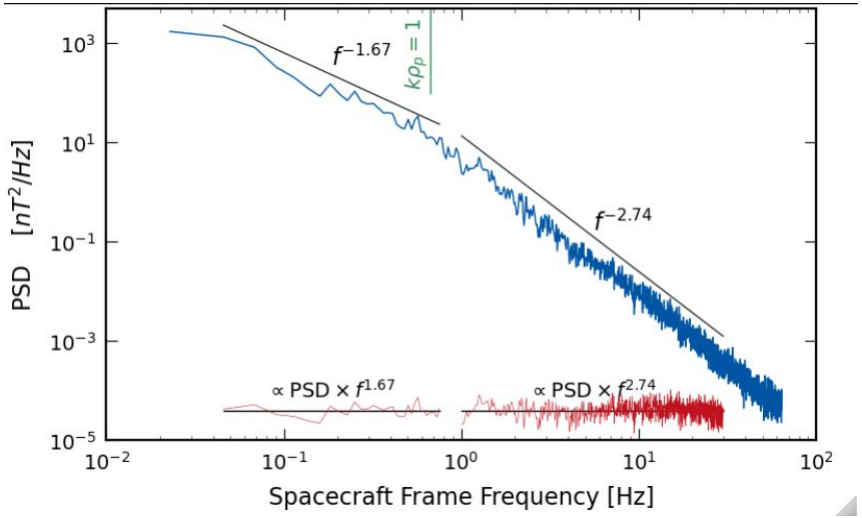
Kinetic Range

Inertial Range



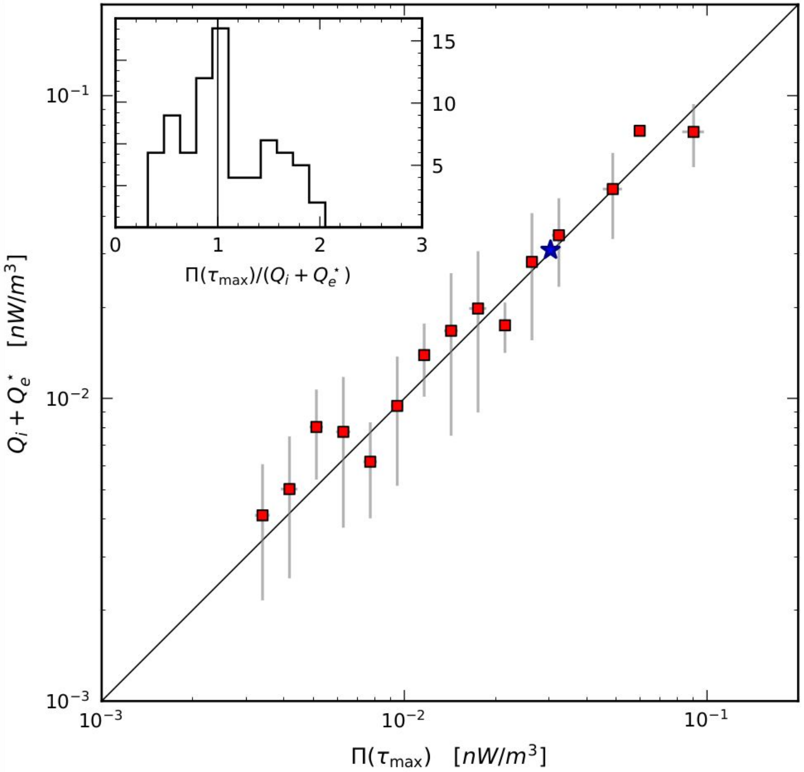
Inertial Range

Kinetic Range



Some statistics ~100 MMS intervals

Is it true in general that $\Pi^{MHD} = Q_i + Q_e$



Turbulence *heats the plasma* via the PS interaction!

At which scales are Ions and Electrons heated?

$$\frac{\partial}{\partial t}\langle \bar{\mathcal{E}}_\alpha^{\mathrm{th}} \rangle = -\overline{P_\alpha} : \nabla \overline{v_\alpha}$$

-PS(ℓ)

Filtered Pressure-Strain interaction

Rate of change of
Large Scale (>ℓ)
Thermal Energy

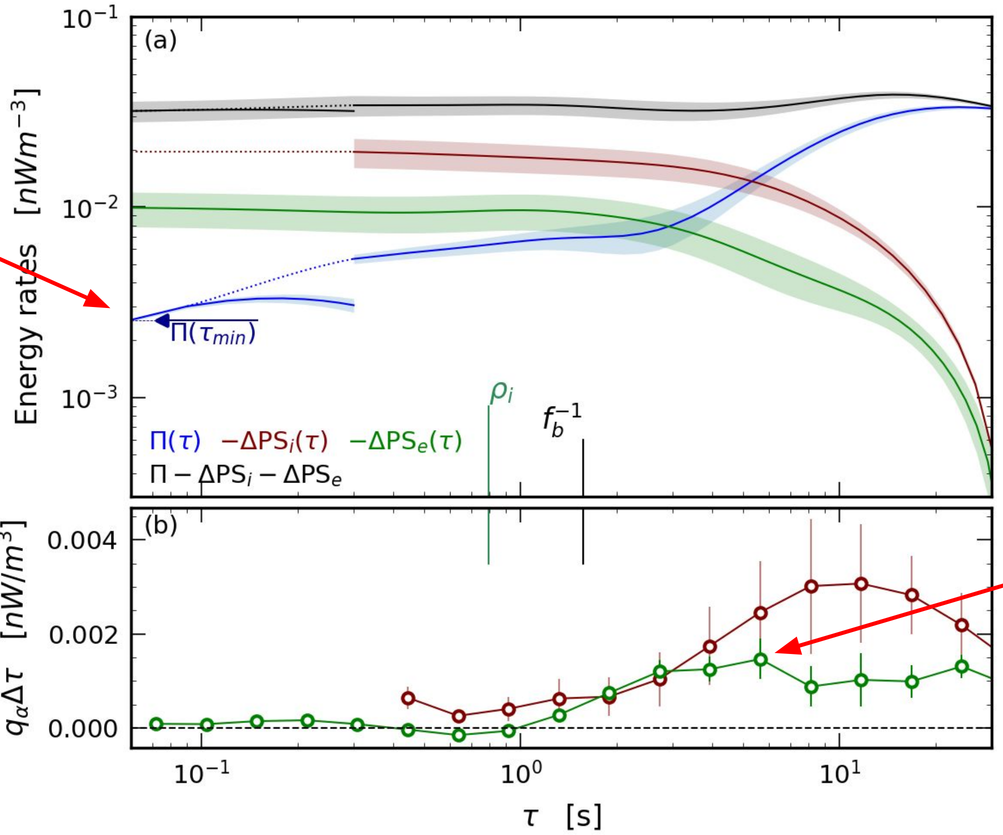
How much “heating” between scale ℓ and ℓ+Δℓ ?

q(ℓ)=-PS(ℓ)+PS(ℓ+Δℓ)

At which scales are Ions and Electrons heated?

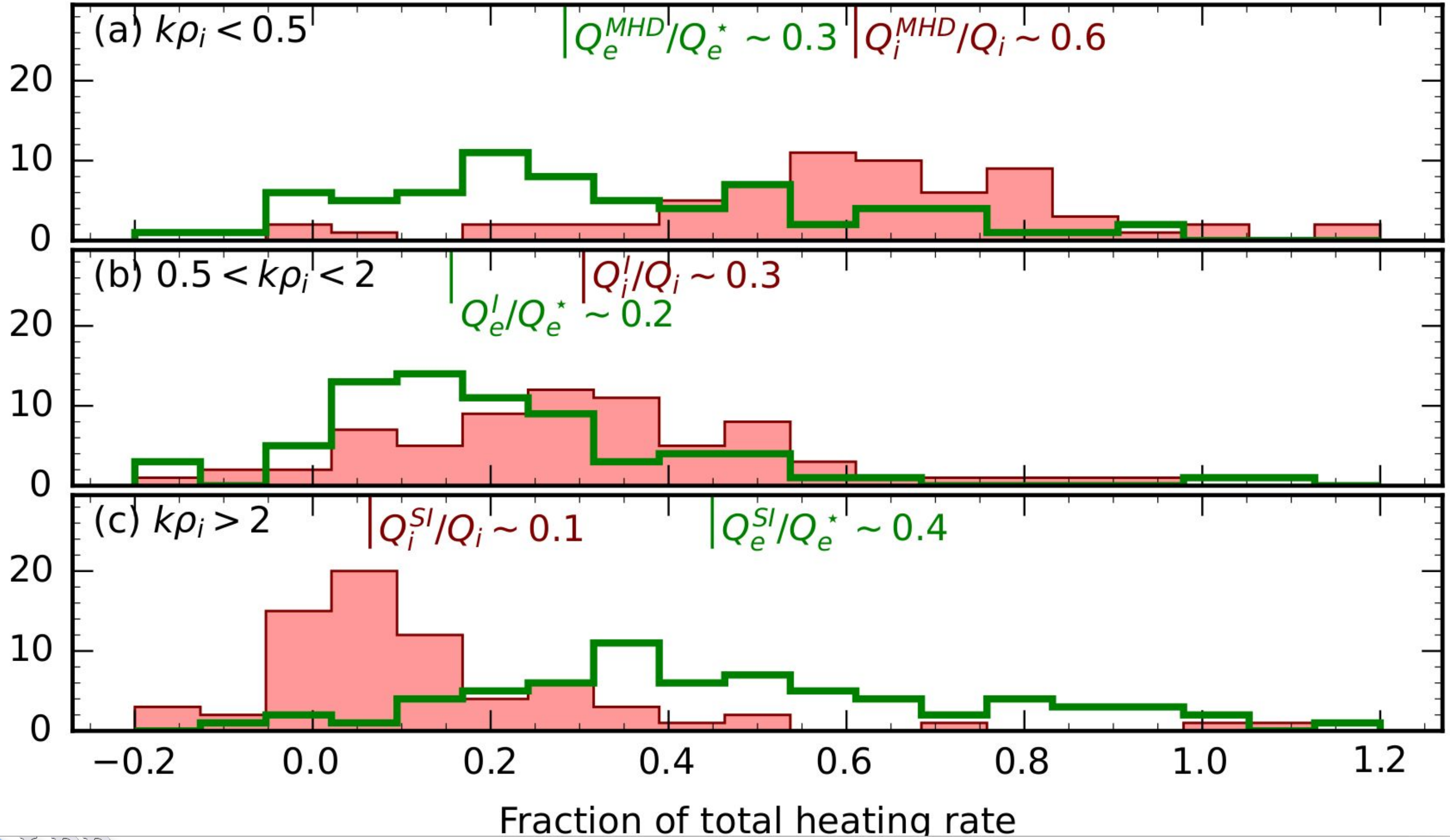
Kinetic Range

Residual Cascade to Electrons



Substantial Electron heating at ion scale $k\rho_i \sim 0.1$

At which scales are Ions and Electrons heated?

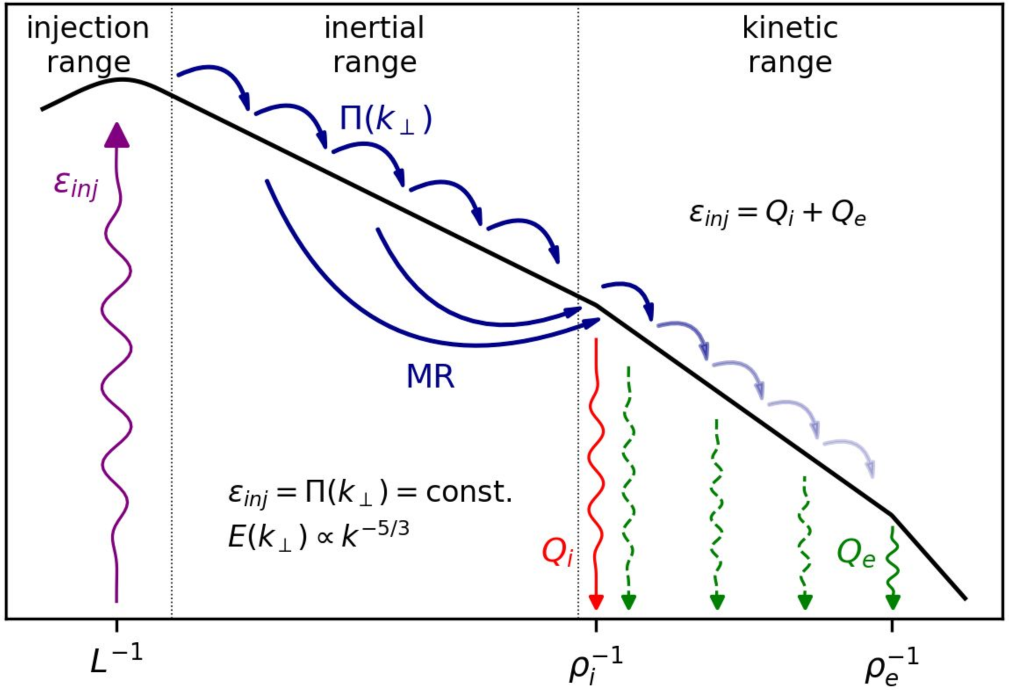


Edge of MHD range

Ion Scale

SubIon Scales

Conclusions Manzini+ PRL 2023, Manzini+ rev. 2024



- The (full) pressure strain plays the role of an *effective dissipation* [Yang+ 2017, Hellinger+ 2022]

- The Kinetic range is (weakly) dissipative and the plasma is heated

- Electron Heating is effective in the full kinetic range